Tectonophysics, 47 (1978) 109—129

ANALYSIS OF FAULTING IN THREE-DIMENSIONAL STRAIN FIELD

ZE'EV RECHES

Department of Geology, Stanford University, Stanford, Calif. 94305 (U.S.A.) *

(Submitted May 16, 1977; revised version accepted June 20, 1977)

ABSTRACT

Reches, Z., 1978. Analysis of faulting in three-dimensional strain field. Tectonophysics, 47: 109—129.

Multiple faults, composed of three, four or more sets of faults, have been observed at a wide range of scales, from clay experiments to rift valleys. Multiple faults usually are explained by multiple phases of deformation. However, in several cases the multiple faults develop *simultaneously;* therefore, they cannot be explained by the common theories of faulting. Furthermore, these theories were derived for plane strain, whereas, multiple faults are associated with three-dimensional strain.

An elementary analysis of faulting in three-dimensional strain field is presented here. The analysis considers the deformation of an idealized model due to slip along sets of faults; the model is subjected to strain boundary conditions. The analysis shows that (1) three or four sets of faults are necessary to accommodate three-dimensional strain, (2) there is a combination of four fault sets which minimize the dissipation of the deformation; the orientation of these faults depend on the strain state, and (3) if the resistance to slip along these four sets of faults is cohesive, then the stresses which cause slippage along them are equal or larger than the yielding stresses of a Tresca rigid-plastic with the same cohesion.

The analysis presented here is too elementary to be directly applied to field observations; however, it indicates that multiple faults and rhomboid patterns of faults probably form when a body is strained three-dimensionally.

INTRODUCTION

One of the most common geologic structures, the fault, which is a surface or narrow zone across which differential shearing displacement is accommodated, still presents some unsolved problems. One of these problems is the origin of multiple sets of faults, comprised of three, four or more sets of faults which almost certainly formed simultaneously (Oertel, 1965; Malone et al. 1975; Aydin, 1977; Reches, 1978).

Simultaneous faulting on multiple sets is inconsistent with explanations of faulting based on stress criteria of failure. Anderson (1951), explained failure of rock by Coulomb's failure criterion, and postulated that faults develop if the stresses on certain planes equal the shear strength of the material. For

* Present address: Geophysical Laboratory, Department of Applied Mathematics, Weizmann Institute of Science, Rehovot, Israel.

frictionless materials, according to Anderson's analysis, there are two conjugate sets of fault planes inclined 45 degrees to the direction of maximum compression and containing the direction of intermediate stress. For frictional materials, the two planes are inclined at somewhat smaller angles to the direction of maximum compression. Odé (1960) and Varnes (1962) extended Anderson's theory of faulting to include descriptions of strains within the material being faulted, which they postulated to behave as a Von Mises, rigid-plastic substance. Even the extended theories of faulting, however, can account for only two sets of faults. According to the plasticity theories of faulting, the traces of faults correspond with characteristic directions within the plastic material. However, these theories require that the material be in a state of plane strain for, otherwise, there are *no* characteristic surfaces in the material (e.g., Thomas, 1952; Craggs, 1954; Odé, 1960). Thus, the standard theories of faulting cannot explain multiple faulting and cannot explain how three-dimensional strains can be accommodated by faulting.

The analysis of faulting presented in the following pages concerns the number of fault sets required to accommodate three-dimensional deformation of a body containing many faults. The analysis is related to those by Taylor (1938) and Bishop (1953), who were concerned with the number of slip systems in metal crystals required for an arbitrary deformation. We will show that for faults with orthorhombic symmetry in a material with cohesive strength, the orientation of faults that minimizes the work done by external loads depends upon the strain to which the body is subjected. The analysis is incomplete because we say nothing about how the faults developed. However, the analysis is complementary to those by Anderson (1951) and Odé (1960). They analyzed faulting under stress boundary conditions whereas we analyze faulting under strain boundary conditions.

First we will review descriptions of multiple faults in the field and in experiments. Then we will derive the number of sets of faults required for two- and three-dimensional strain. Finally, we will relate the orientations of faults to the state of strain to which an idealized body is subjected.

EXAMPLES OF MULTIPLE FAULTS

Faults have similar patterns in a wide variety of scales and tectonic settings. For example, rhomboid patterns of fault traces are common in the Basin and Range Province, in margins of rift valleys, as well as in cakes of clay. Donath (1962) described faults in the Basin and Range Province in south-central Oregon (Fig. 1). He indicated that most of the faults dip steeply, that some are nearly vertical, and that both strike-slip and dip-slip displacements can be recognized on some of the fault surface. Thompson and Burke (1974) described the pattern of faults in the Basin and Ranges as follows:

"Individual faults tend to be extremely crooked in map plan and the fault pattern is more nearly rhomboid or even rectilinear. Some mountain ranges

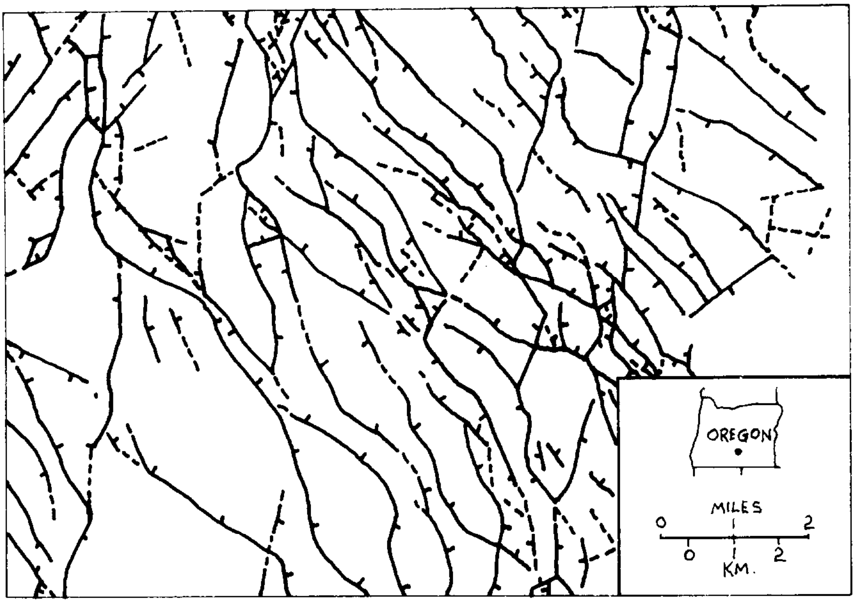

Fig. 1. Rhomboid pattern of Cenozoic normal faults in south-central Oregon. Bars on down thrown side of faults; fault dashed where inferred (from southeast portion of plate 3, Donath 1962).

are bounded by echelon faults that strike diagonal to the range. . . Nowhere is the fault pattern better exhibited than in the late Cenozoic basalt flows of south-central Oregon (Fig. 1, here), but similar patterns are common from Nevada to Texas. Moreover, the roughly rhomboid map pattern of faulting is characteristic of other regions of present or past crustal extension, such as the African rifts, the Rhine graben, the Oslo graben, and the Triassic basins of eastern North America."

Thompson and Burke (1974) presented several hypotheses to explain the patterns, including changes in stress with time, influence of older structures and anisotropy. They concluded, however, that the mechanics of the faulting must be complex.

Oertel (1965) apparently first recognized the simultaneous development of multiple faults. He described and analyzed small faults developed in experiments with cakes of clay subjected to three-dimensional deformation. He deformed a cake of clay, 9 cm thick, by extending or shortening the cake in one direction (X_1 in Fig. 2). The top and sides of the cake were free surfaces, so the cake was subjected to mixed stress and displacement boundary conditions. Careful strain measurement indicated a three-dimensional strain field, with almost constant volume in the clay. Four sets of faults with orthorhombic symmetry formed in both extension and compression experiments (Fig. 2). Oertel noted that *none* of his main observations, including the number of sets, the orientations and the slip directions of the faults, could be explained

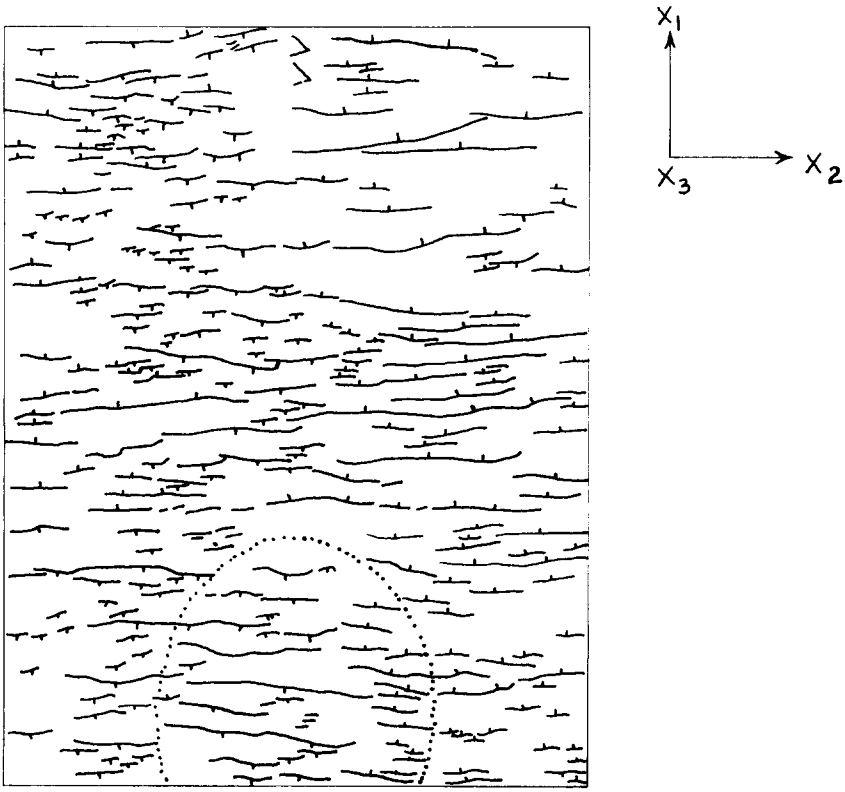

Fig. 2. Faults in clay cake at the end of elongation experiments. View on top of the cake; short diameter of ellipse 6.2 cm, bars on down thrown side (drawn from Oertel, 1965, fig. 6).

in terms of theories of faulting based on failure criteria, including Anderson's (1951). To explain the fault pattern developed in his experiments, Oertel suggested that the *interference* between the various fault sets controls the geometry of the pattern.

Robin and Currie (1971) described small faults or joints in metavolcanic rocks north of Madoc, Canada. They found five sets of fractures with similar appearance (Fig. 3). Field evidence indicates that slip occurred primarily along three sets of fractures, that the fourth set accommodated minor slip and that the fifth set generally accommodated no slip (Fig. 3). Robin and Currie (1971) suggested that the five sets of fractures developed as joints in response to nonuniform stresses and that shear offsets occurred along the fractures later.

Reches (1977) reported closely spaced faults in sandstone in the Palisades monocline, Grand Canyon, Arizona (Fig. 4). Four sets of faults with orthorhombic symmetry can be recognized, so the fault pattern is similar to those

Fig. 3. Joint orientation in metavolcanics, Madoc, Canada. The contours indicate five sets of joints marked *A* to *E*. Lower hemisphere, equal area net (from Robin and Currie, 1971).

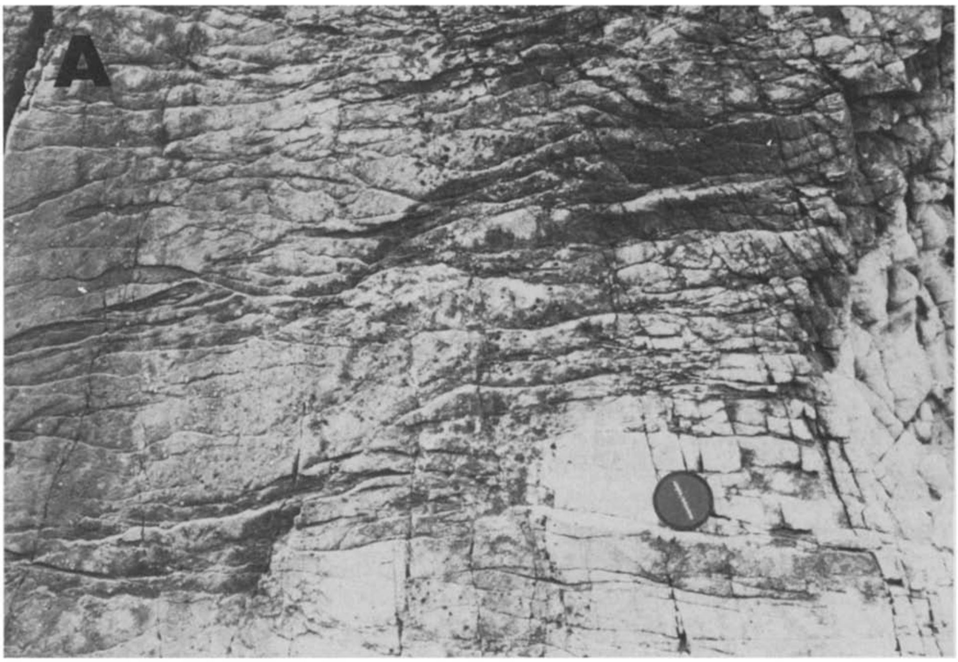

Fig. 4A.

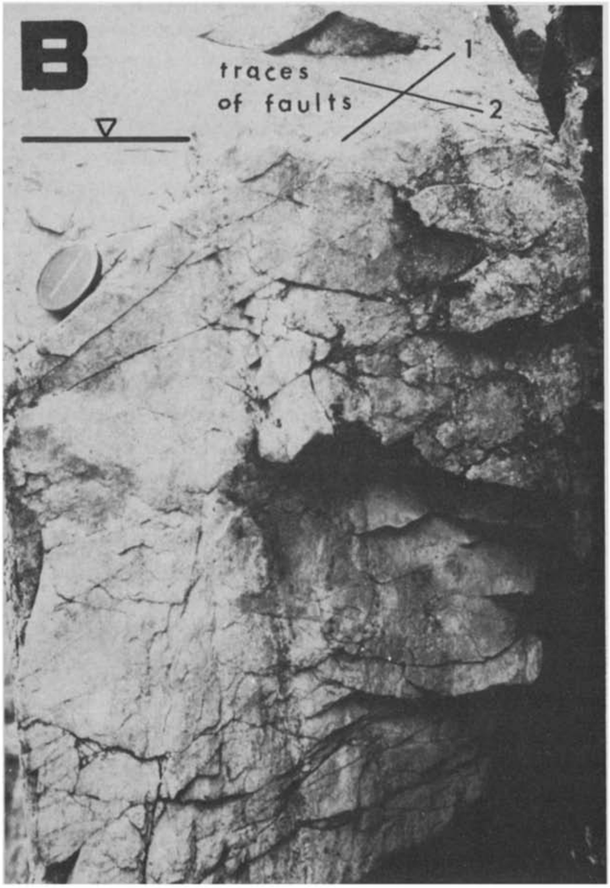

Fig. 4. Closely spaced faults in a steeply dipping sandstone layer, Palisades Creek, Arizona. A. View on top of layer. B. Side view of same layer (from Reches, 1978).

developed in Oertel's clay-cake experiments (Fig. 2).

Aydin (1977) has observed multiple sets of faults in sandstones at numerous localities in the San Rafael Desert, Arches National Park, and the Henry Mountains, Utah. Cross-cutting relations of members of the various sets indicate that the sets formed simultaneously. Offsets across the faults indicate oblique slip so the faults accommodated three-dimensional strains.

Malone et al. (1975) detected three faults in a micro-earthquake swarm in Columbia River Basalt in Eltopia, eastern Washington. Using records of 42 events, they located three fault planes and derived the orientation of the principal stress axes from focal mechanisms (Fig. 5). The maximum compression axes associated with the earthquakes along the three faults nearly coincide (Malone et al., 1975, fig. 14), however, the inferred compression

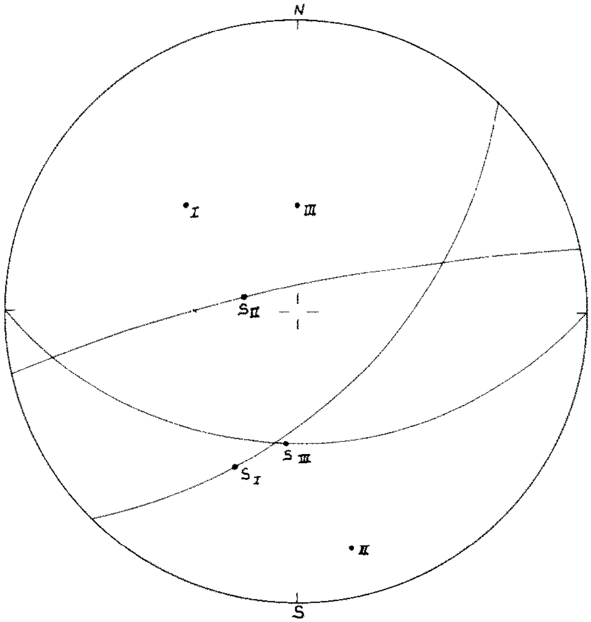

Fig. 5. Faults and slip directions in microearthquake swarm, Eltopia, Washington. Lower hemisphere, stereographic net. [(from Malone et al., 1975).] *I*, *II* and *III* are poles to the three faults, S_I, S_{II}, and S_{III} are slip directions on the three faults.

axis is parallel to the bisector of the obtuse angle between two of the three faults.

In each of these examples there are three or more sets of faults rather than the two sets predicted by theories of faulting based on failure criteria, such as Anderson's (1951), or on plasticity theory. Furthermore, in all examples where gross strain of the rock or clay is known, the strain was three dimensional, rather than plane strain, as required in the theories of faulting based on plasticity theories. The standard explanation of multiple sets of faults is that each pair of sets of failure formed separately; during each deformation one pair of sets of faults formed, and according to this interpretation, multiple faulting implies multiple deformation. However, multiple deformations can be excluded for each of the examples where the deformation history is known, as in the experiments by Oertel (1965), the earthquake swarm in Washington (Malone et al., 1975), and the faults in the San Rafael Desert, Utah (Aydin, 1977).

An explanation for the simultaneous development of three or more sets of faults is needed. We shall discuss some aspects of multiple faulting in the following pages.

THE MODEL

Slip along faults in response to three-dimensional deformation of the body will be analyzed for a relatively simple model. The model has the following properties:

(1) Deformation is solely by slip along sets of faults, resulting in simple shear parallel to each set (App. I., Fig. 10).

(2) There is a sufficient density of faults in each set so that the deformation of a body containing the faults can be approximated by that of a homogeneous material.

(3) The resistance to slip along the faults is cohesive, independent of the normal stress.

The model is sufficiently simple so that solutions can be obtained in closed form, and the results can be readily understood. Furthermore, as the analysis considers strains to be infinitesimal, other types of deformation such as elastic, plastic or viscous deformation of the blocks between the faults can be superimposed on the deformations produced by faulting.

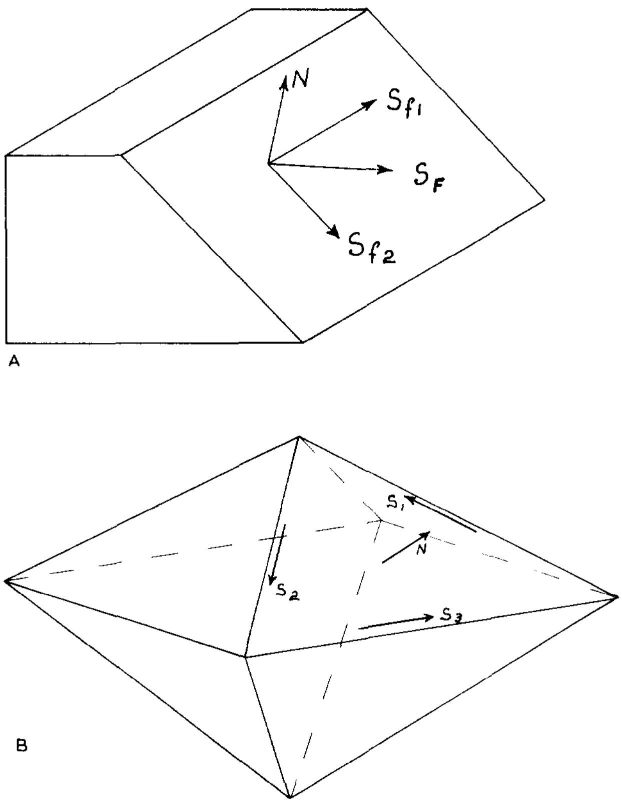

Fig. 6. A. Slip on a fault plane. S_F is the slip direction, S_{f1} and S_{f2} are two independent slip directions. The total slip in S_F direction can be represented by linear combination of slips on S_{f1} and S_{f2} directions. B. Slip systems on an octahedral face of aluminum crystal. N is the normal to the face, S_1, S_2 and S_3 are three possible slip directions.

NUMBER OF FAULT SETS REQUIRED FOR TWO- AND THREE-DIMENSIONAL DEFORMATIONS

Let us consider the number of sets of faults needed to accommodate general, three-dimensional deformation of the ideal model described above. The analysis partly follows those by Taylor (1938) and by Oertel (1965). First, however, we should clarify the difference between a *fault* and a *shear system*. The slip direction on a fault, S_F (Fig. 6A), can have any orientation within the plane of the fault; this direction can be represented by linear combination of two independent slip directions, S_{f1} and S_{f2} (Fig. 6A). We will define a *shear system* as an independent slip direction, S_{f1} or S_{f2}, on a fault (Fig. 6A). As the slip within a fault plane can be in an arbitrary orientation, S_F, then each fault corresponds to *two* shear systems. Shear systems are kinematically equivalent to slip systems in crystals (e.g., Taylor, 1938) (Fig. 6B).

The general infinitesimal deformation in three dimensions is:

$$D_{ij} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \tag{1}$$

which can be separated into strain E_{ij}, and rotation ω_{ij}:

$$D_{ij} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} + \begin{bmatrix} 0 & \omega_{12} & \omega_{13} \\ \omega_{21} & 0 & \omega_{23} \\ \omega_{31} & \omega_{32} & 0 \end{bmatrix} \tag{2}$$

where X_1, X_2 and X_3 are cartesian coordinates. For constant volume deformation, which is required if deformation is solely due to simple shear along faults:

$$D_{33} = -(D_{11} + D_{22}) \tag{3}$$

Thus, there are eight independent components in the deformation tensor eq. 1. The deformation components can be written as the sum of contributions of deformation from k shear systems:

$$D_{ij} = \sum_{n=1}^{k} D_{ij}^{n} \qquad \begin{matrix} i = 1, 2, 3 \\ j = 1, 2, 3 \end{matrix} \tag{4}$$

where D_{ij}^{n} is the contribution of the n^{th} shear system to the total deformation of the material. Eq. 4 can be written as:

$$D_{ij} = \sum_{n=1}^{k} T_{ij}^{n} \cdot \gamma_n \qquad \begin{matrix} i = 1, 2, 3 \\ j = 1, 2, 3 \end{matrix} \tag{5}$$

where γ_n is the amount of simple shear along the n^{th} shear system, and T_{ij}^{n} is the coefficient for transformation from the coordinates of the shear system

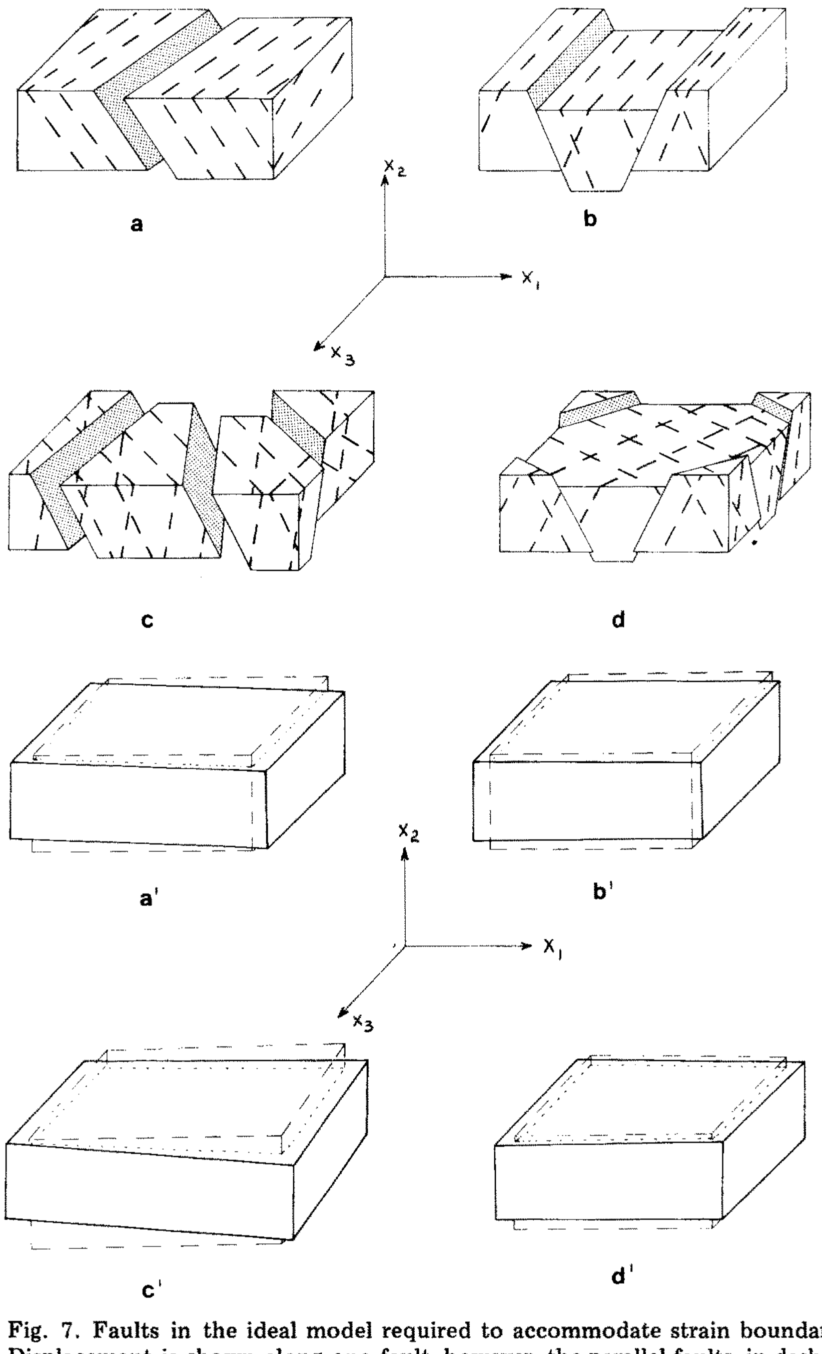

Fig. 7. Faults in the ideal model required to accommodate strain boundary conditions. Displacement is shown along one fault, however, the parallel faults, in dashed lines, carry similar displacements. Figures a′ to d′ show the deformation of an initial rectangular parallelopiped (dashed lines), by slip along faults, into a parallelopiped (solid line). a. One component of the strain is specified and one fault is required (see text). Note the rotation and shear of the boundaries of the model. b. Plane strain with three specified components of strain. Note the lack of rotation and shear of boundaries of the model. c. Three-dimension strain by slip along three faults. Note the rotation and shear of boundaries of the model. d. Three-dimensional deformation with no rotation or shear of the boundaries of model.

to the general coordinates. Eqs. 5 are a system of eight linear equations. The vector γ can be considered to be the unknown and the T_{ij} to be the known matrix of coefficients:

$$[T_{ij}] \cdot [\gamma_n] = D_{ij} \tag{6}$$

To obtain a unique solution for γ, there should be eight terms, not all zero, in each equation. This implies that *eight shear systems are sufficient to accommodate a general deformation in three dimensions.* Further, because each fault corresponds to two shear systems, *four sets of faults are sufficient to accommodate a general deformation in three dimensions.*

However, if less than eight components of the deformation are applied, then fewer shear systems will be sufficient to accommodate the deformation. Let us demonstrate this conclusion by means of several examples.

Suppose that plane strain is applied to the body, and that only one component of the applied deformation is specified (Fig. 7a). For example:

$$D_{11} = D$$

where D is a constant. According to eq. 5 only one shear system of arbitrary orientation is required to accommodate the strain field. Figure 7a indicates that indeed one system can satisfy the condition. However, it is clear that both shear and rotation result from the slip along a single shear system (Fig. 7a).

Suppose that the body is again subjected to plane strain, but that the applied deformation is such that X_1 and X_2 are the principal strain axes, and no rotation of the boundaries of the body is permitted (Fig. 7b). Thus:

$$D_{11} = E_1; D_{12} = 0; D_{21} = 0$$

where E_1 is the principal strain, and $D_{22} = -D_{11}$. Therefore, three shear systems or two sets of faults of arbitrary orientation are sufficient to accommodate this plane strain. It should be noted that two sets of faults correspond to *four* shear systems and, therefore, one shear system is redundant.

Similarly, for three-dimensional strain one can distinguish between cases in which rotation is unspecified (Fig. 7c), which is analogous to the first example above, and cases in which rotations are specified (Fig. 7d), which is analogous to the second example above. If we specify only the *strain* part of the general deformation (eq. 6), five shear systems (or three faults) are needed to accommodate the strain, as concluded by Taylor (1938). Again, it should be noted that three sets of faults include a redundant shear system. If we specify both the rotations and the strains, for example if the material is confined by rigid boundaries that cannot rotate (Fig. 7d), then eight shear systems, or four sets of faults, of arbitrary orientations, are sufficient to accommodate the deformation.

PREFERRED SETS OF FAULTS

The analysis of the number of sets of faults and shear systems required to satisfy three-dimensional deformation is quite general. The orientations of the fault sets required are arbitrary. We can predict orientations of faults in simple materials if we make three further assumptions about the material: Let us assume that the faults within the material yield if the shear stress acting on them is equal to a constant, say C. That is, the condition

$$|\tau| = C \tag{7}$$

must be satisfied on each fault set for that set to slip and to contribute to the deformation of the body. We could assume that the shear stress required to activate a set of faults depends upon the normal stress on those faults, as in Coulomb's law, but this would complicate the analysis unnecessarily for this preliminary study of faulting in three dimensions. The second assumption we will make is that the faults that will be activated are those that minimize the dissipation of the deformation, that is, that minimize the work done by external forces. Similarly, Taylor (1938) postulated that aluminum crystals deform along slip systems which minimize the dissipation of energy. The sets of faults which minimize the dissipation we will call the *preferred faults.* Finally, we will assume that the principal stress axes coincide with the principal strain axes.

The ideal model of a faulted body contains many closely spaced faults so, as indicated in Appendix I, the dissipation associated with slip on one set is:

$$W = C\gamma_n \tag{8}$$

in which γ_n is the simple shear across the set. The total dissipation for the four sets of faults is the sum:

$$W^t = C \sum_{n=1}^{4} \gamma_n \tag{9}$$

Because the dissipation depends only on the total simple shear (eq. 9), the sets of faults which minimize the simple shear also minimize the dissipation.

However, we can deduce some important information from the yield condition stated in eq. 7. The combination of the assumption that the shear stress along the sets of faults is equal and that of stress and strain states are symmetric requires that activated faults be symmetrical about the principal stress axes. For general, three-dimensional deformation, in which there are four sets of faults, the faults must have orthorhombic symmetry. Furthermore, if the axes of principal stress and strain coincide, as was assumed above, the simple shear along each set of faults will be the same, γ.

The magnitude of the simple shear for each of the four sets of faults with orthorhombic symmetry, according to derivations in Appendix II, is:

$$\gamma^* = \left[\frac{\frac{1-N_1^2}{N_2^2} + k^2\frac{1-N_2^2}{N_1^2} + 2k}{1-N_1^2-N_2^2}\right]^{1/2} \tag{10a}$$

where: $k = E_2/E_1$ (10b)

$\gamma^* = \gamma/E_1$ (10c)

$E_3 = -(E_1 + E_2)$ (10d)

and E_1, E_2 and E_3 are principal strains. Here $N_1 = \cos^{-1}\theta_1$, where θ_1 is the angle between the normal to fault set and the axis X_1, and $N_2 = \cos^{-1}\theta_2$ where θ_2 is the angle between the normal to the same fault set and axis X_2.

The simultaneous solution of the derivatives $\partial\gamma^*/\partial N_1 = 0$ and $\partial\gamma^*/\partial N_2 = 0$ provides the direction cosines, N_1 and N_2, of the fault sets which minimize the dissipation. The solutions are:

$$\begin{aligned} N_1 &= \pm(1/2)^{1/2} & S_1 &= \pm N_1 \\ N_2 &= \pm(-k/2)^{1/2} & S_2 &= \pm N_2 \\ N_3 &= \pm\{(1+k)/2\}^{1/2} & S_3 &= \pm N_3 \end{aligned} \tag{11a}$$

for $-1 \leqslant k \leqslant 0$, and:

$$\begin{aligned} N_1 &= \pm 1/2(1+k)^{1/2} & S_1 &= \pm N_1 \\ N_2 &= \pm\{k/2(1+k)\}^{1/2} & S_2 &= \pm N_2 \\ N_3 &= \pm(1/2)^{1/2} & S_3 &= \pm N_3 \end{aligned} \tag{11b}$$

for $0 \leqslant k \leqslant 1$ where $k = E_2/E_1$. The strain ratio, k, may have any value from $-\infty$ to $+\infty$. For simplicity we show in eqs. 11 only the solutions for the range $-1 \leqslant k \leqslant 1$ which implies $E_1 \geqslant E_2 \geqslant E_3$. Solutions for the full range of k are plotted in Fig. 8.

The orientations of the preferred faults are functions of the ratio of the principal strains applied to the ideal body. The orientations of these faults are plotted for several strain ratios in Figs. 8 and 9.

Figure 8 shows the orientations of the normals to the preferred sets of faults with respect to the principal strain axes. Note that there are two preferred sets for plane strain: $k = -1$, $k = 0$, or $k = \pm\infty$, whereas there are four sets of faults for the three-dimensional strain. Figure 9 shows both the sets and the corresponding slip directions in their relative orientations. Again, for plane strain: $k = -1$ and $k = 0$, there are two fault sets with either strike-slip or dip-slip, respectively, whereas four fault sets appear for three-dimensional strains, with either oblique- or dip-slip.

We can also derive an expression for the principal stresses required to cause slippage along the four sets of preferred faults. According to Jaeger and Cook (1969, p. 21), the shear stress, τ, on an arbitrary plane is:

$$\tau^2 = (\sigma_1 - \sigma_2)^2 N_1^2 N_2^2 + (\sigma_2 - \sigma_3)^2 N_2^2 N_3^2 + (\sigma_3 - \sigma_1)^2 N_3^2 N_1^2 \quad (12)$$

where σ_1, σ_2 and σ_3 are the principal stresses. Substituting eqs. 7 and 11 into eq. 12:

$$-(\sigma_1 - \sigma_2)^2 k - (\sigma_2 - \sigma_3)^2 k(k+1) + (\sigma_3 - \sigma_1)^2 (k+1) - 4C^2 = 0 \quad (13a)$$

for $-1 \leqslant k \leqslant 0$, and:

$$(\sigma_1 - \sigma_2)^2 k/(k+1) + (\sigma_2 - \sigma_3)^2 k + (\sigma_3 - \sigma_1)^2 - 4(k+1)\,C^2 = 0 \quad (13b)$$

for $0 \leqslant k \leqslant 1$.

Equations 13 are the stress conditions which are required for faulting of the ideal model. For example, in plane strain, $k = 0$, the stresses are:

$$|\sigma_1 - \sigma_3| - 2C = 0 \quad (14a)$$

which is the yield condition for a Tresca material. In another example, the ideal material is subjected to compression in a triaxial test, $k = -0.5$, and $\sigma_2 = \sigma_3$, so the stresses required for faulting are:

$$|\sigma_1 - \sigma_3| - 2C = 0 \quad (14b)$$

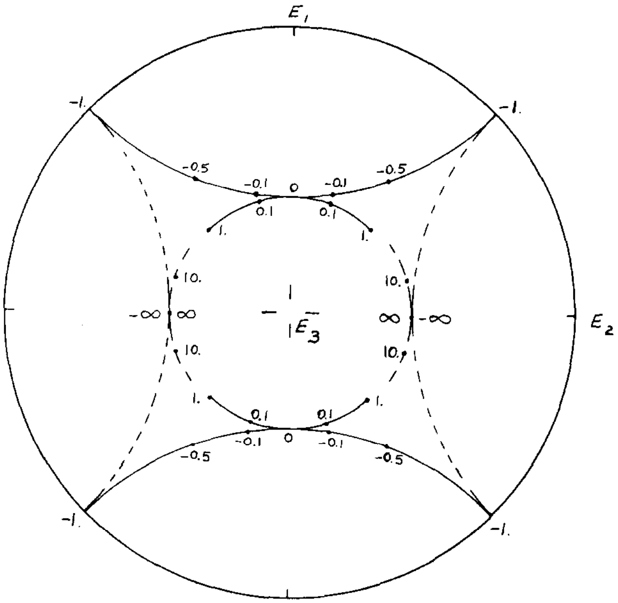

Fig. 8. Poles to preferred faults in the three-dimensional strain for various values of k, X_1, X_2, and X_3 are the principal strain axes, k is the ratio of the principal strains, E_2 to E_1. Solid lines refer to solutions in eqs. 11, broken lines to solution for the complete range of k.

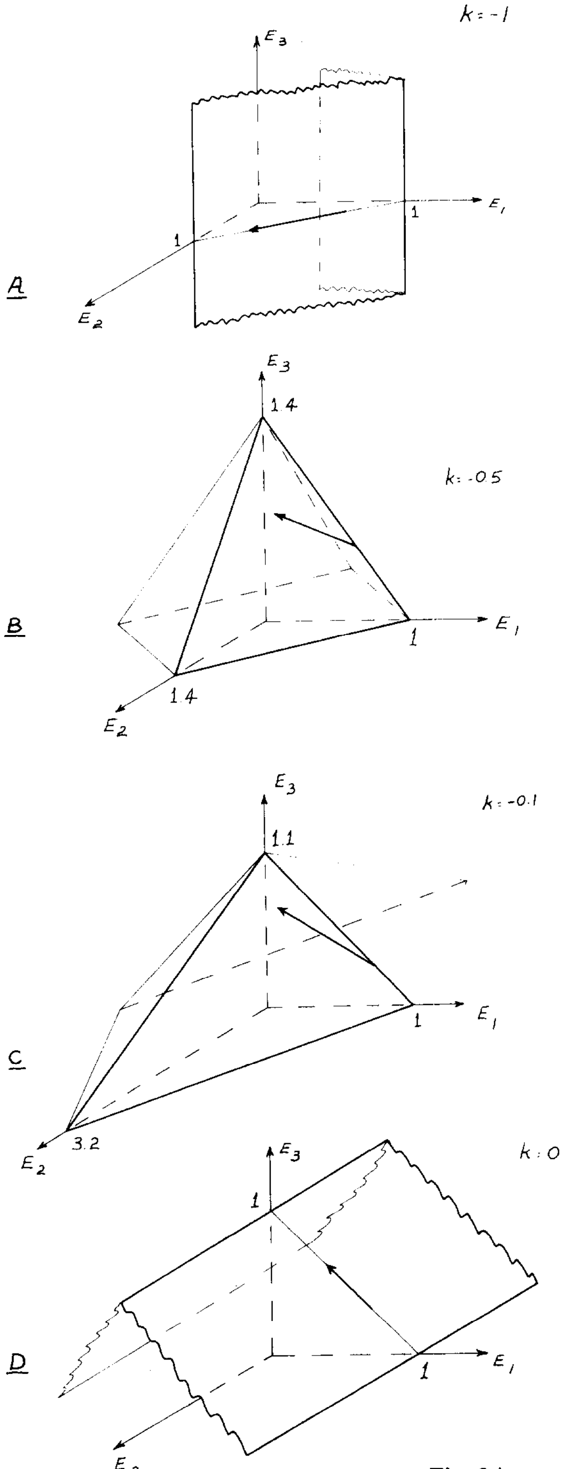
k = -1
E3
E1
E2
1
1
A
E3
1.4
k = -0.5
E1
1
1.4
E2
B
E3
k = -0.1
1.1
E1
1
3.2
E2
C
E3
k : 0
1
E1
1
E2
D

Fig. 9A.

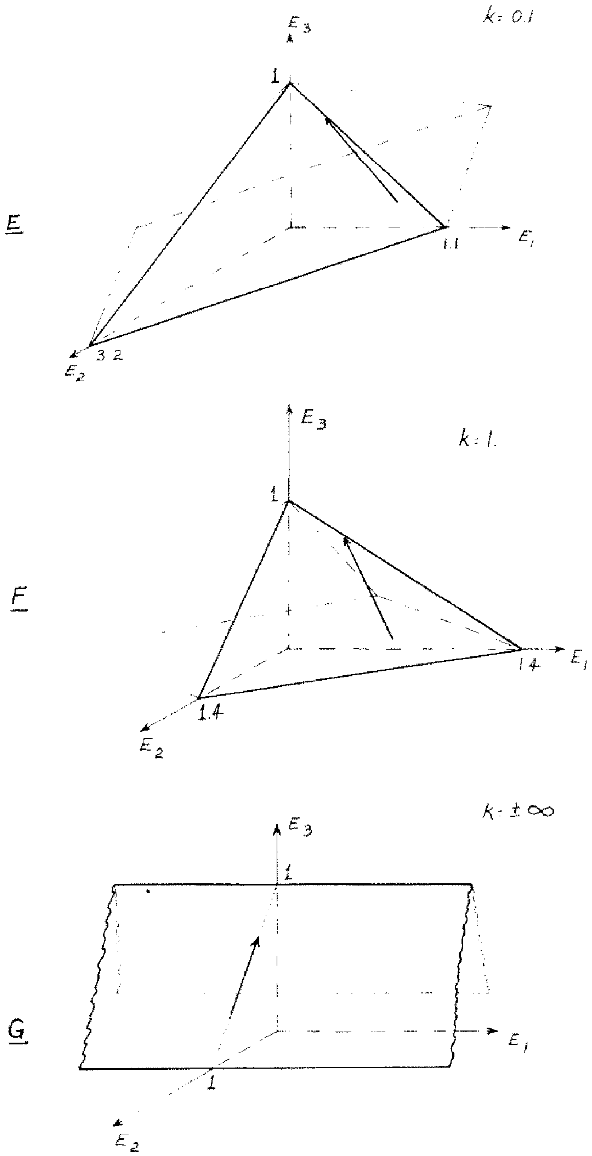

Fig. 9. Preferred faults and slip orientations for various values of strain ratio k. The faults are shown as triangles intersecting the principal strain axes. Note the coordinates of intersection with the axes. Heavy arrows indicate slip direction.

as in plane strain. However, for extension in which $k = 1$ and $\sigma_1 = \sigma_2$, the stresses required for faulting are:

$$|\sigma_1 - \sigma_3| - 2\sqrt{2}C = 0 \qquad (14c)$$

Equations 13 and 14 show that the stresses required for faulting of the ideal model, subjected to *strain* boundary conditions, are equal to or larger than the stresses required to deform the ideal rigid-plastic, Tresca substance with the same yield strength, C.

CONCLUSION

We analyzed the faulting of an idealized model which accommodates strain solely by slip along sets of faults. The model contains many faults in each set, and the resistance to slip along the faults is cohesive. We derived three results for the faulting of this model under infinitesimal strain boundary conditions.

(1) Three sets of faults of arbitrary orientation are sufficient to accommodate three-dimensional strain. If a rotation field is applied on the model, in addition to the strain field, then four sets of faults of arbitrary orientation are sufficient to accommodate the deformation. It was also shown that two sets of faults are sufficient for plane strain cases.

By making two assumptions, first, that the activated faults in the model minimize the dissipation of the deformation, and second, that the principal strain axes coincide with the principal stress axes, we derived two additional results:

(2) Four sets of preferred faults in orthorhombic symmetry are activated during faulting of the model. Furthermore, the orientation of the preferred sets depends only on the state of strain.

(3) The stresses required to cause slippage along the preferred sets of faults with cohesion C, are equal to or larger than the stresses required to deform an ideal rigid-plastic Tresca substance with the same cohesion C.

The three results of our analysis, concerning the number of sets of faults, the orientations of the preferred faults and the stresses required for faulting, are different from the corresponding theoretical results of Odé's (1960) and Anderson's (1951) analyses. Furthermore, we cannot compare our analysis with any other faulting analysis because, to the best of our knowledge, other analyses were derived for plane strain only. Oertel's analysis (1965) is an exception. It was derived for three-dimensional strain, but it is incomplete and predicts the number of sets of faults only for the irrotational case and does not predict the orientations of the preferred faults.

The analysis presented here is too simple to be directly applied to specific field or experimental observations. Even so, it provides some insight into the patterns of faults described in the introduction. Many of these patterns have orthorhombic symmetry (Figs. 2—5), and some carry oblique slip (Figs. 2 and 3). According to our analysis, such patterns could accommodate three-dimensional strain, and they probably formed when such a strain was imposed. Further development of our elementary theory may lead to a better explanation for these fault patterns.

ACKNOWLEDGEMENTS

I thank Ray C. Fletcher and Arvid M. Johnson of Stanford University, and Gerhard Oertel of U.C.L.A., for stimulating discussions on faulting problems. Arvid M. Johnson, Ray C. Fletcher and Paul Delaney critically reviewed early versions of this paper and made many useful comments.

This study was supported by the National Science Foundation Grants No. EAR 75-15849 and EAR 76-03273.

APPENDIX I

The objective of this appendix is to derive the dissipation energy due to slip along a single set of faults. Let us consider a plate with width b which is normal to a set of faults (Fig. 10). The energy dissipated due to slip U_p along a single fault is:

$$w = U_p \cdot |\tau| \cdot A \tag{A1}$$

where $|\tau|$ is the shear stress along the fault and A is the surface area of fault. In our model the shear stress equals the cohesion C (eq. 7). The average dis-

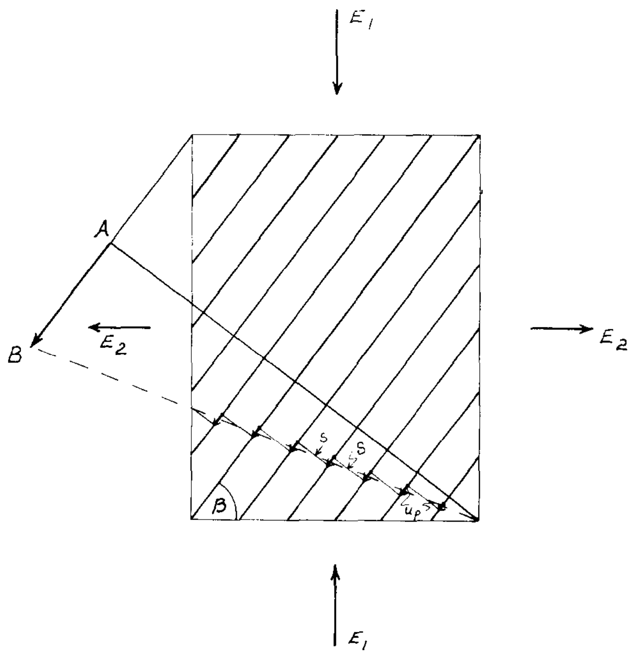

Fig. 10. Simple shear associated with slip along discrete faults in a set. The set makes an angle β with the plane normal to E_1, the maximum strain axis. S is spacing, U_p average slip along a fault.

placement on a single fault in a set is (Fig. 10):

$$\dot{U}_p = \gamma \cdot S \quad \text{(A2)}$$

where S is the spacing of the faults and γ is the simple shear of the set. Substituting eqs. 7 and A2 into eq. A1, for the total surface area of all faults in the set (Fig. 10):

$$w^* = C \cdot \gamma \cdot S \cdot L \cdot b \quad \text{(A3)}$$

where L is the total length of the faults in the plate and b is the width of the plate. The volume of the plate is:

$$V = S \cdot L \cdot b \quad \text{(A4)}$$

So the dissipation energy of one set of faults per unit volume becomes:

$$W = C \cdot \gamma \quad \text{(A5)}$$

APPENDIX II

The objective of this appendix is to derive the orientation of faults which minimize the simple shear along them. We consider only deformation due to simple shear along sets of faults. The general deformation in the coordinates of the set of faults is:

$$d_{ij} = \gamma \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{(A7)}$$

where γ is the simple shear along the set (Fig. 10). The coordinates are: X_1 normal to the fault plane, X_2 normal to $X_1 - X_3$ plane, and X_3 parallel to the slip direction. The standard transformation of deformation from the coordinate system of the set to the general coordinate system is:

$$D^n_{ij} = a_{ik} a_{jl} d_{kl} \quad \text{(A8)}$$

where D^n_{ij} is the contribution of the n^{th} set of faults to the general deformation, and a_{ik} and a_{jl} are the direction cosines between the two coordinate systems. Substituting eq. A7 into eq. A8 yields:

$$D^n_{ij} = \gamma_n \begin{bmatrix} N_1S_1 & N_1S_2 & N_1S_3 \\ N_2S_1 & N_2S_2 & N_2S_3 \\ N_3S_1 & N_3S_2 & N_3S_3 \end{bmatrix} \quad \text{(A9)}$$

where $N_i = a_{pi}$ are the direction cosines of the normal to the fault and $S_i = a_{pi}$ are the direction cosines of the slip direction. The total deformation of the model is obtained by substituting eq. A9 into eq. 4.

Let us now consider the strain due to simple shear along four sets of faults in orthorhombic symmetry. The symmetry axes of the sets of faults are the principal strain axes and the shear along the four sets is equal:

$$\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma \tag{A10}$$

From the symmetry of the fault pattern, one obtains that:

$$\begin{aligned} &N_1^1 = N_1^2 = -N_1^3 = -N_1^4 = N_1 \qquad && S_1^1 = S_1^2 = -S_1^3 = -S_1^4 = S_1 \\ &N_2^1 = -N_2^2 = N_2^3 = -N_2^4 = N_2 && S_2^1 = -S_2^2 = S_2^3 = -S_2^4 = S_2 \\ &N_3^1 = N_3^2 = N_3^3 = N_3^4 = N_3 && S_3^1 = S_3^2 = S_3^3 = S_3^4 = S_3 \end{aligned} \tag{A11}$$

where N_i^n are direction cosines of the normal to the n^{th} fault plane and S_i^n are the direction cosines of the slip direction. Substituting eqs. A9, A10, and A11 into eq. 4 yields:

$$E_{ij} = 4\gamma \begin{bmatrix} N_1S_1 & 0 & 0 \\ 0 & N_2S_2 & 0 \\ 0 & 0 & N_3S_3 \end{bmatrix} \tag{A12}$$

which is the strain tensor as function of the orientation of four sets of faults in orthorhombic symmetry.

From eq. A12:

$$\begin{aligned} E_{11} &= 4\gamma N_1 S_1 \\ E_{22} &= 4\gamma N_2 S_2 \end{aligned} \tag{A13}$$

We know that:

$$N_1S_1 + N_2S_2 + N_3S_3 = 0$$

or:

$$N_1S_1 + N_2S_2 + (1 - N_1^2 - N_2^2)^{1/2}(1 - S_1^2 - S_2^2)^{1/2} = 0 \tag{A14}$$

because the slip direction is in the fault plane. Substituting eqs. A13 into eq. A14, rearranging and squaring both sides of the equation yields eq. 10a in the text.

REFERENCES

Anderson, E.M., 1951. The Dynamics of Faulting. Oliver and Boyd, Edinburgh, 2nd ed., 191 pp.

Aydin, A., 1977. Faulting in Sandstone, Utah. Ph.D. Diss., Stanford, Calif.

Bishop, J.F.W., 1953. A theoretical examination of the plastic deformation of crystals by glide. Philos. Mag., 44: 51—64.

Craggs, J.W., 1954. Characteristic surfaces in ideal plasticity in three dimensions. Q.J. Mech. Appl. Math., 7 (1): 35—39.

Donath, F.A., 1962. Analysis of basin-range structure, South-central Oregon. Geol. Soc. Am. Bull., 73: 1—16.

Jaeger, J.C. and Cook, N.G.W., 1969. Fundamentals of Rock Mechanics. Methuen, London, 515 pp.

Malone, S.D., Rothe, G.H. and Smith, S.W., 1975. Details of microearthquake swarms in the Columbia Basin, Washington. Seismol. Soc. Am. Bull., 65: 855—864.

Odé, H., 1960. Faulting as a velocity discontinuity in plastic deformation. Geol. Soc. Am., Mem., 79: 293—321.

Oertel, G., 1965. The mechanism of faulting in clay experiments. Tectonophysics, 2: 343—393.

Reches, Z., 1978. The development of monoclines. Part I: Structure of the Palisades Creek branch of the East Kaibab monocline, Grand Canyon, Arizona. In: V. Matthews, (Editor), Laramide Folding Associated with Basement Block Faulting in the Western U.S. Geol. Soc. Am., Mem. (in press).

Robin, P.-Y. and Currie, J.B., 1971. Analysis of fracturing in Precambrian volcanics north of Madoc, Ontario. Can. J. Earth Sci., 8: 1302—1313.

Taylor, G.I., 1938. Plastic strain in metals. J. Inst. Met., 62: 307—324.

Thomas, T.Y., 1952. On the characteristic surfaces of the Von Mises plasticity equations. J. Ration. Mech. Anal., 1: 343—357.

Thompson, G.A. and Burke, D.B., 1974. Regional geophysics of the Basin and Range Province. Annu. Rev. Earth Planet. Sci., 2: 213—238.

Varnes, D.J., 1962. Analysis of plastic deformation according to Von Mises' theory with application to the south Silverton area, San Juan County, Colorado. U.S. Geol. Surv., Prof. Pap., 378-B: 49 pp.