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DEFORMATION OF A FOLIATED MEDIUM

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ABSTRACT

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The deformation of a medium which contains a single family of parallel planes of weakness such as faults or foliation, is analyzed for plane strain conditions. It is shown that if slip along the planes of weakness and rotation of these planes are the main mechanisms of deformation, the mode of yielding of the medium depends mostly on the angle θ which the planes of weakness make with the maximum compressive stress. The medium yields in an unstable manner with a marked drop in stress for $\theta < 45^\circ - \phi/2$, where ϕ is the friction angle. The medium yields stably with work hardening for $\theta \geqslant 45^\circ - \phi/2$.

The predictions of this analysis are in agreement with the experimental results of deformation of Martinsburg slate by Donath (1964, 1968).

INTRODUCTION

The deformation of foliated or jointed rock as well as the deformation of large faulted regions is affected by preexisting planes of weakness which produce anisotropic behaviour in the deforming medium. Commonly *slip* along these planes of weakness and *rotation* of the regions bounded by the planes of weakness are the dominant mechanisms of deformation of the foliated or faulted medium. In plate tectonics theory, for example, we consider the plates to be rigid, and attribute the global deformation to displacements along narrow zones. Also kink bands form in multilayers where slip and rotation of the layers occur with negligible internal deformation (e.g., Paterson and Weiss, 1966; Donath, 1968; Reches and Johnson, 1976).

Usually, in the analysis of geologic structures, we apply equations which were derived for infinitesimal strain, to structures which underwent finite strain. In so doing we neglect effects of rotation and nonlinearities, for simplicity. In the present analysis, a simple model is presented for faulted or foliated material which takes into account rotation and is valid for finite strain.

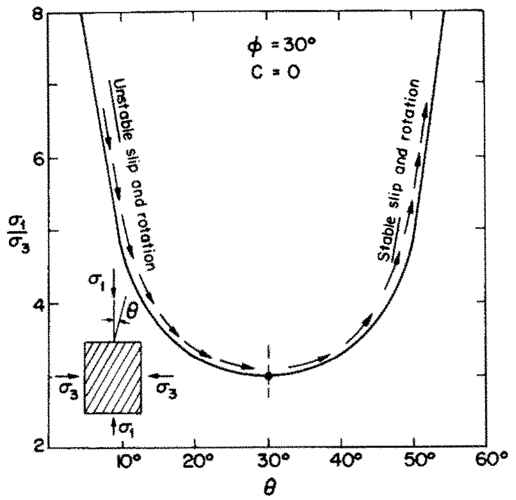

Fig. 1. Relation between the ratio of the principal stresses and the slope angle of planes of weakness. The resistance to slip is frictional, with θ the angle of friction. Arrows indicate changes in stress-ratio during slip and rotation of the planes of weakness (after Reches and Johnson, 1976).

Reches and Johnson (1976), who introduced the model, analyzed the behaviour of a multilayer which is subjected to stresses in which the principal direction of greatest compression is inclined at an angle θ to layering (Fig. 1). If the contacts between the layers are frictional, then the medium will yield unstably due to slip along the layering. By unstable yielding we mean that the stress difference necessary to continue slippage is smaller than the stress difference necessary to initiate slippage (Fig. 1). Therefore, a stress drop, which depends on the angle θ, is associated with kinking.

In this paper, the case of unstable yielding by shear is extended to a general model of deformation of faulted or foliated media. It is shown using a few examples that the deformation of a faulted medium is well explained by the rheological behaviour derived here.

THE RHEOLOGY OF A FOLIATED MEDIUM

Let us consider a body of foliated material; the contact strength between layers is both cohesional, with cohesion C, and frictional, with friction angle ϕ. The layers are isotropic and homogeneous, and are inclined at an angle θ to the maximum compressive stress σ_1 (Fig. 1). We will assume the stresses within the body are uniform and satisfy static equilibrium. We do not specify the nature of the foliation and the analysis is applicable to slates, simple multilayers, schists, jointed layers and faulted regions. The equations governing the deformation of the foliated medium depend primarily on geometric relationships and are almost independent of the rheological behaviour of the layers.

The stresses are assumed to be homogeneous and therefore the stresses on a plane of weakness can be derived from Mohr's circle for plane strain conditions. The planes of weakness contain the axis of intermediate stress, σ_2. During the initiation of slippage along the planes of weakness the stresses are (e.g., Jaeger and Cook, 1968, p. 66):

$$\sigma_1 - \sigma_3 = \frac{2C + 2\sigma_3 \tan \phi}{(1 - \tan \phi \tan \theta) \sin 2\theta} \tag{1}$$

where σ_1 and σ_3 are the maximum and minimum compressive stresses, respectively, C and ϕ are the cohesion and angle of friction along the planes of weakness, and θ is the angle between the plane of weakness and the axis of maximum compression.

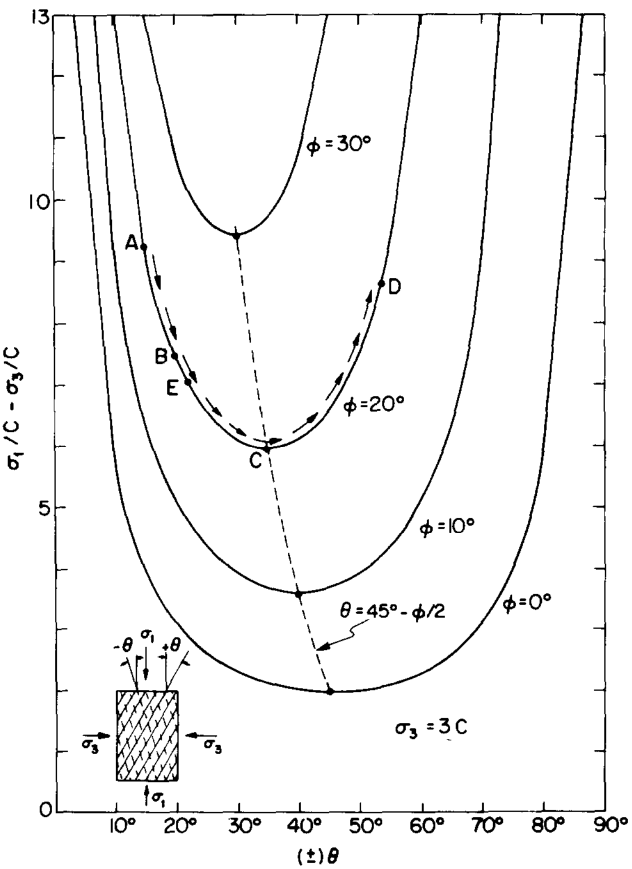

Fig. 2. Relation between the stress difference and the inclination of a foliated body for the case of σ_3 three times the cohesion C. Curves were drawn for various friction angles ϕ.

Rewriting eq. 1 yields:

$$\sigma_1/C - \sigma_3/C = \frac{2 + 2P \cdot \tan\phi}{(1 - \tan\phi \tan\theta)\sin 2\theta} \tag{2}$$

where $P = \sigma_3/C$. Figure 2 shows eq. 2 plotted for a few angles of friction and for $P = 3$. Any value of P results in the formation of similar curves. Suppose that a foliated medium with $\phi = 20°$ and $\theta_i = 15°$ is subjected to a least stress $\sigma_3 = 3C$. Then, at yielding, according to eq. 2:

$$\sigma_1/C - \sigma_3/C = 9.3$$

which is point A in Fig. 2. If some of the boundaries of the foliated medium permit rotation (Fig. 3), the planes of weakness may rotate to a new θ, say

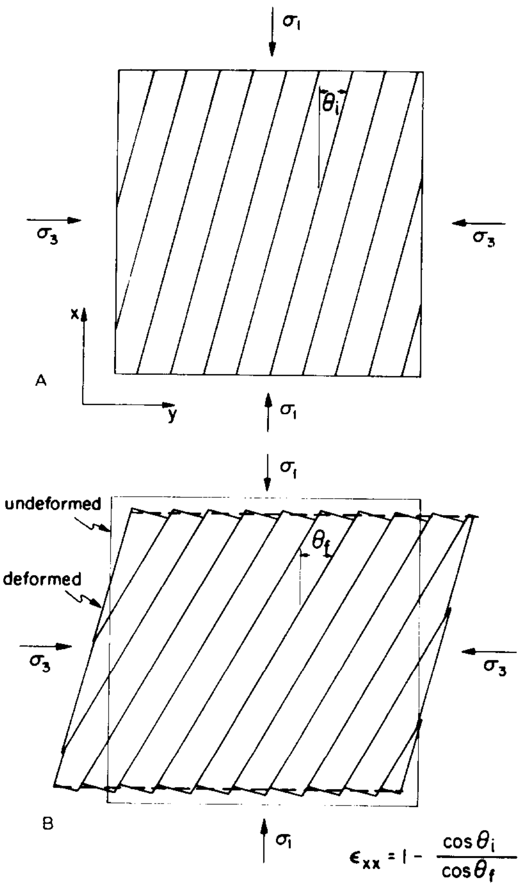

Fig. 3. The strain of a foliated material. A. Undeformed state with initial inclination θ_i of the planes of weakness. B. Deformed state with final inclination θ_f. Note the rotation of the sides of the model.

$\theta_f = 20°$ (B, Fig. 2). However, for $\theta = 20°$, the stress difference necessary for slippage is:

$$\sigma_1/C - \sigma_3/C = 7.5$$

which is smaller than the stress difference for $\theta = 15°$ (A, Fig. 2). Therefore, if the planes of weakness are inclined at $\theta < (45° - \phi/2)$, the yielding is unstable. Once slippage starts, it continues with a significant stress drop (path A to C, Fig. 2). If $\theta \geqslant 45° - \phi/2$, the stress difference must be increased in order to maintain slippage, and the process of increasing the slip becomes stable (path C to D, Fig. 2). (For further details see Reches and Johnson, 1976.)

The strain of a foliated medium is the sum of the continuous strain inside the layers, ϵ^P, and the discontinuous strain due to slip and rotation of the faults ϵ^f:

$$\epsilon = \epsilon^P + \epsilon^f$$

In the present analysis we assume that the layers are perfectly rigid and therefore, that ϵ^P vanishes. The material is termed a rigid foliated medium.

Consider the plane strain of a rigid foliated medium inclined in angle θ to σ_{xx} and let σ_{xx} be the maximum compressive stress σ_1, and σ_{yy} be the minimum compressive stress σ_3 (Fig. 3). Examine especially the case in which the medium is bound by rigid, non-rotating plates normal to the x-axis and is not bound on the sides normal to the y-axis (Fig. 3). These coordinates are selected to make the foliated body looks in a triaxial apparatus, where the rigid plates prevent rotation of the ends of the specimen, but the jacket allows rotation of the sides of a specimen. The strain due to slip and rotation can be expressed as:

$$\epsilon_{xx} = 1 - \frac{\cos \theta_i}{\cos \theta_f} \tag{3}$$

where θ_i and θ_f are the initial and final inclinations of planes of weakness (Fig. 3) (e.g., Donath, 1968; Freund, 1974). It should be noted that both ϵ_{xx} and γ may be finite.

A strain—stress relationship can be derived through eqs. 1 and 3 in the following steps:

(1) For an initial inclination, θ_i of foliation and a given strain ϵ_{xx}, calculate the final inclination θ_f (equation 3). For example, for $\theta_i = 15°$, $\epsilon_{xx} = -1\%$ (shortening), $\theta_f = 17°$.

(2) Calculate the stress difference, $\sigma_1 - \sigma_3$ necessary to maintain slippage at an inclination of θ_f by means of eq. 1. The calculated stress difference for $\theta_f = 17°$ for example, corresponds to a strain $\epsilon_{xx} = -1\%$.

Figure 4 shows the stress—strain curves for a block of Martinsburg slate tested by Donath (1968). The values of θ and C for Martinsburg slate are calculated similarly to Donath (1961). These stress—strain curves are valid only for post failure behaviour of the slate.

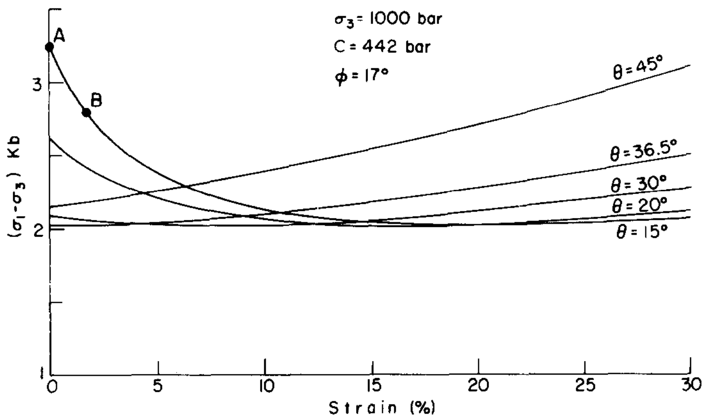

Fig. 4. Theoretical strain—stress curves for Martinsburg slate (Donath, 1968) calculated by means of eqs. 1 and 3 for confining pressure of 1000 bars. Cohesion is 442 bars and friction angle is 17°.

The strain—stress curves of $\theta_i = 15°$, $\theta_i = 36.5°$ and $\theta_i = 45°$ in Fig. 4 may serve as example for all other curves. The curve for $\theta_i = 15°$ shows an ultimate strength of:

$$(\sigma_1 - \sigma_3)_{15°} = 3258 \text{ bars}$$

at a least stress of 1000 bars. If the stress difference is increased until the ultimate strength is reached, the slate yields unstably with a *stress drop* to 2383 bars at 5% shortening. The stress difference continues to decrease up to a strain of 20%, when the inclination of the foliation is 36.5°, and the stress difference is 2021 bars. For further deformation the stress difference must again be increased for every increment of strain. Therefore, the first 20% of strain occur *unstably*, with stress drop, whereas strain above 20% is stable, with increase of stress.

At an initial inclination of $\theta_i = 36.5°$, the critical stress difference is at a minimum, $\theta_{min} = 45° - \phi/2$, so the ultimate strength is:

$$(\sigma_1 - \sigma_3)_{min} = 2021 \text{ bars},$$

for $\sigma_3 = 1000$ bars. The stress difference increases with increasing strain, and therefore the deformation occurs *stably*. There are no stress drops, and the strain—stress curve is of a "work hardening" type.

The ultimate strength for $\theta_i = 45°$ is 2154 bars for $\sigma_3 = 1000$ bars, more than for $\theta_i = 36.5°$. The "work hardening" effect is stronger than for $\theta_i = 36.5°$ as manifested by the steeper strain—stress curve.

In summary, for inclinations smaller than $45° - \phi/2$ the yielding behaviour is unstable and there is an associated stress-drop; for inclinations larger than or equal to $45° - \phi/2$, yielding is stable, with stress increasing. Figure 4 was drawn for a specific material and one confining pressure, however, curves for other materials and confining pressures can be obtained easily by means of eqs. 1 and 3.

DEFORMATION OF MARTINSBURG SLATE

Deformation by slip and rotation was studied previously by Freund (1974) and Garfunkel (1974), and it was mentioned by investigators of kink bands such as Donath (1964, 1968) and Reches and Johnson (1976). Probably the most comprehensive experimental studies of slip and rotation in a foliated rock has been made by Donath (1961, 1964, 1968, 1969) who deformed two blocks of Martinsburg slate in triaxial compression. Cores were taken at 0°, 15°, 30°, 45°, 60°, 75° and 90° to the foliation of the slate, and were tested at confining pressures upto 2000 bars. Here, we will discuss a few aspects of Donath's results.

Donath (1964, p. 289) distinguished between the mode of deformation of samples oriented 30°—90° and the mode of deformation of samples oriented 15°. Samples oriented 30° or more failed by slip on foliation planes and later by development of a fault zone. The slip occurred when a yielding differential stress was reached; however, slip ceased for differential stresses lower than the yielding stress. Also: "In other tests, each increment of displacement on the fault required an additional increment of differential stress (work hardening)" (Donath, 1964, p. 289).

On the other hand, samples oriented at 15° failed with a marked drop of the differential stress (Fig. 6). In tests up to 1000 bars confining pressure, the minimum difference reached during the initial stress drop was sufficient to maintain slip and rotation on several cleavage planes without the development of kink-bands (Donath, 1964, Fig. 7). In tests at confining pressure of 1000 bars or higher, an increment of strain beyond the minimum stress difference (Fig. 4) required an additional increment of stress (work hardening). Furthermore, kink bands developed during these tests (Fig. 5). These modes of deformation described by Donath can be explained as a normal behaviour of a foliated medium presented above.

First, the analysis predicts that a foliated material with an angle of friction of 17°, as is the case in Martinsburg slate, should yield unstably for inclinations $\theta_i < 36.5°$. The observed stress—strain curve for $30° < \theta_i < 36.5°$ is almost identical to the curve of $\theta_i = 36.5°$ (Fig. 3). Therefore, for samples with a θ_i larger than 30°, deformation is a "work hardening" process without stress drop, and this is confirmed by Donath's observations.

Second, the predicted stress—strain curve for $\theta_i = 15°$ (Fig. 4) shows no work hardening upto a strain of more than 20%. The work hardening observed in Martinsburg slate (Fig. 5) is a result of the development of kink-bands because the intensive rotation of the layers in the kink band require additional stress for further strain. Work hardening did not occur in tests of $\theta_i = 15°$ which did not develop kink-bands such as the tests at the relatively low confining pressures of less than 1000 bars (Donath, 1964, Fig. 6).

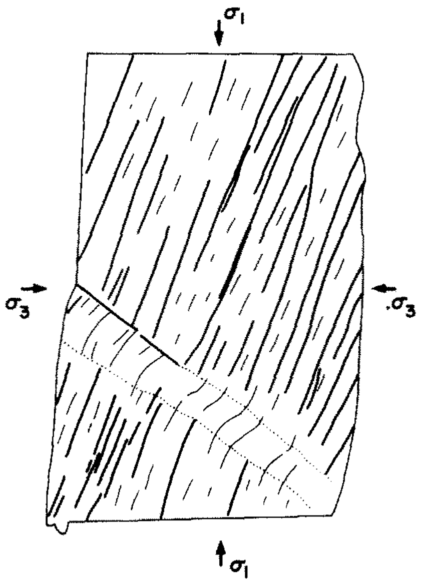

Fig. 5. Deformed specimen of Martinsburg slate initially inclined 15° to σ_1. Thin short lines are cleavage planes, thick lines are gouge zones developed on cleavage planes. Note the 5° general rotation of the cleavage planes to 20°, the rotation of the sides of the sample (compare with Fig. 3), and the unrotated planes normal to σ_1 (redrawn from Donath, 1968).

The present predictions of the behaviour of a foliated medium are in general consistent with what Donath found to be the mode of faulting in Martinsburg slate. Let us now examine some tests in detail.

The experiments in which the inclinations were at 15° to the axis of maximum compressive stress were described in most detail by Donath (1968, 1969). He notes three stages of deformation: (1) elastic deformation of the intact specimen; (2) general slip on many folia; and (3) local yielding which formed a narrow kink-band (Fig. 5). The last two processes are two modes of unstable yielding; local yielding follows general slip because of local inclination of the folia. For example, a foliated material with $\phi = 20°$ and $\theta_i = 15°$ (point A, Fig. 2) yields by general slip to $\theta_f = 20°$ (point B, Fig. 2). Slip on a local perturbation with inclination of $\theta_l = 22°$ (point E, Fig. 2), requires a smaller stress difference than the general slip. Therefore, a kink band may be formed at this local perturbation.

Yielding and the onset of slip occur together with a sharp stress drop. Donath (1969) suggested, according to his strain calculations, that the kinking occurs during the stress-drop following yielding. The rotation inside the kink-band contributes to the general deformation of the sample (Donath, 1968, eq. 3). According to Donath (1968, p. 277), the general rotation immediately after yielding, and before kinking, is 3°—5°. The post-yielding

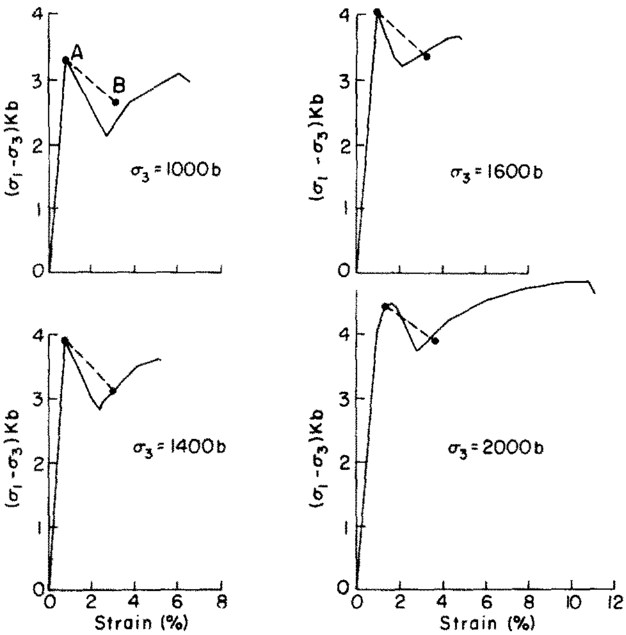

Fig. 6. Comparison of experimental strain—stress curves of Martinsburg slate with those of a theoretical strain—stress relation of a foliated material. Continuous lines are experimental results (after Donath, 1968). Dashed lines are the theoretical strain—stress curves calculated for general rotation of 4°, cohesion of 442 bars and friction angle 17°.

theoretical strain-stress curves for samples of $\theta_i = 15°$, are plotted in Fig. 6 together with the experimental curves. For example, the line *AB* in Fig. 6A is part *AB* of the complete strain—stress curve in Fig. 4. The comparison of the theoretical and experimental curves (Fig. 6) indicates one general feature: for all confining pressures, the theoretical stress drop is smaller than the observed one, and the theoretical total strain is larger than observed. This indicates that the kinking, producing a zone of larger than average rotation, may have occurred during the stress drop, and may have caused an additional stress drop associated with the rotation in excess of that associated with the 4° rotation of the general foliation outside the kink-band. On the other hand, the total strain contributed by the single kink-band is small, because its width is a small fraction of the length of the sample (Donath, 1968, eq. 3). Therefore, we conclude, in accordance with Donath (1969), that kinking occurs during the first stress-drop and enhances it.

The results of Donath's series of experiments, including strain-stress relationships, ultimate strength values, stress-drop phenomena for cores with $\theta_i < 45° - \phi/2$, strain hardening for cores with $\theta_i \geqslant 45° - \phi/2$, are well explained by the present analysis of strain and stress in a foliated medium.

CONCLUSIONS

A material which contains a single family of parallel planes of a weakness, and which undergoes deformation primarily by slip and rotation of these planes is studied here. The planes of weakness could be bedding planes, foliation, cleavage, joints or faults. The analysis of a special case, a foliated medium with rigid blocks, is based on geometric configuration alone, and the derived stress—strain relationships depend on friction only.

The stress—strain curves of such a medium can be compared with the stress—strain curves of elastic, viscous and plastic materials (Fig. 7). It is evident that the foliated body behaves differently. The characteristic features of the foliated medium are:

(1) Unstable yielding for cases of initial inclination of the folia to maximum compressive stress smaller than 45° $-\theta/2$, where θ is the friction angle.

(2) Stable yielding for cases of initial inclination larger than or equal to 45° $-\theta/2$.

(3) Significant rotation of the planes of weakness as well as rotation of the boundaries of the model (Fig. 3). The present approach explains the well documented experimental results of Donath (1964, 1968, 1969) and those of Gay and Weiss (1974) (detailed discussion: Reches and Johnson, 1976).

The behaviour of the foliated medium derived here is a special case of the general behaviour of an anisotropic, rigid, Coulomb-plastic material. A complete derivation for such material would predict the characteristic features mentioned above for the foliated medium. To the best of our knowledge, a theory for such anisotropic plastic material has not been published. Therefore, the results of the present analysis can be applied to geologic structures even though the complete theory is yet to be derived.

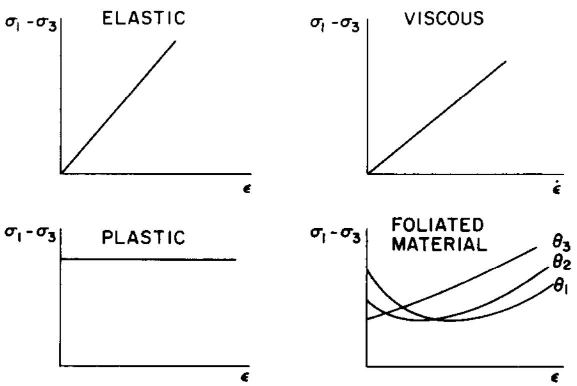

Fig. 7. Comparison of strain—stress curves of ideal elastic, viscous, plastic and foliated material. θ_1, θ_2 and θ_3 are the initial inclinations of the foliation. θ_3 corresponds to the critical angle and it equals 45° $-\phi/2$.

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